

MP3, Grade 4

Task: Joey's Run

Practice standard focus: MP3. *Construct viable arguments and critique the reasoning of others.* Mathematically proficient students at the elementary grades construct mathematical arguments—that is, explain the reasoning underlying a strategy, solution, or conjecture—using concrete referents such as objects, drawings, diagrams, and actions. . . . Mathematically proficient students can listen to or read the arguments of others, decide whether they make sense, ask useful questions to clarify or improve the arguments, and build on those arguments.

Content standard focus: 4.NF 3. Understand a fraction a/b with $a > 1$ as a sum of fractions $1/b$. 4. NF 4. Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.

Introduction

A fourth-grade class discusses two methods for solving the following problem:
Joey runs $\frac{3}{4}$ of a mile each day for 12 days. How many miles does Joey run in all?

The class has been working on a series of story problems that involve multiplying a whole number times a fraction. The day prior to the video, the class worked on these three problems.

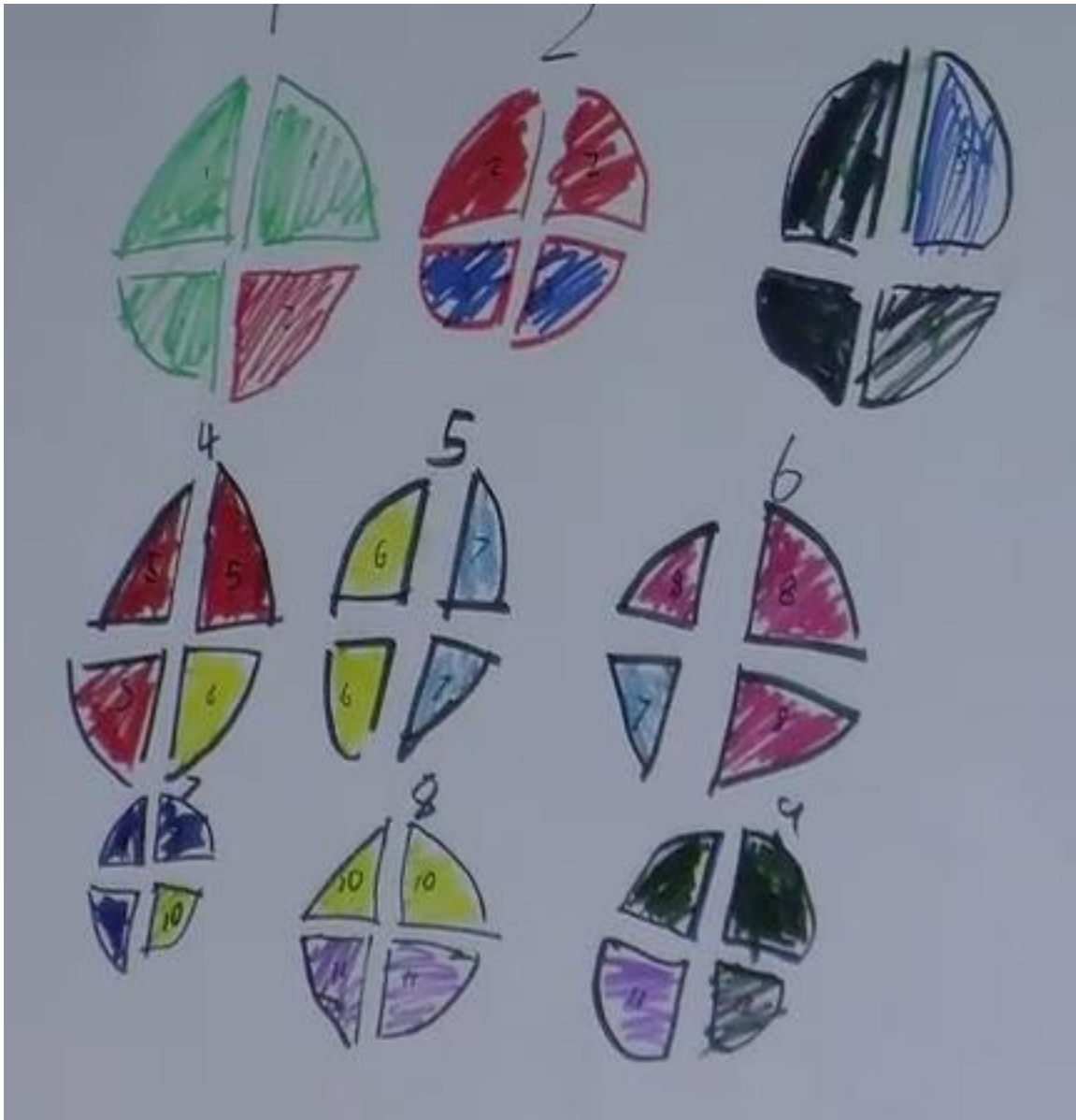
Maya runs $\frac{1}{2}$ of a mile each day for 12 days. How many miles does Maya run in all?

K.C. runs $\frac{1}{4}$ of a mile each day for 12 days. How many miles does K.C. run in all?

Joey runs $\frac{3}{4}$ of a mile each day for 12 days. How many miles does Joey run in all?

The teacher asked some of her students to copy their work onto poster paper so that the class could discuss their reasoning.

On the day of the video, the class first discusses Jay and Harry's representation. Before looking at the video, take a moment to study this representation and explain to yourself how it can be used to solve the third problem about how far Joey runs.



After the students work to understand Jay and Harry's thinking, they turn to Isabel's work. Isabel used her answer to the first problem, about how far Maya runs, to solve the second problem, about how far K.C. runs, to solve the problem about how far Joey runs. How would you use the first two problems to solve the third?

Video : Joey's Run, Part 1¹ [\[**Link\]](#)

Video : Joey's Run, Part 2² [\[**Link\]](#)

¹ Unpublished video from the project, *Using Routines as an Instructional Tool for Developing Elementary Students' Conceptions of Proof*. © TERC, 2013. Used with permission. All rights reserved.

Commentary

This lesson is an example of MP3: *Construct viable arguments and critique the reasoning of others*. In the video, the class considers two arguments for $12 \times \frac{3}{4} = 9$. Everyone in the class has already solved the problem, each with his or her own strategy, and they all know the correct answer is 9 miles. The purpose of the lesson is to understand the reasoning of their classmates.

In the first part, Jay and Harry's argument is largely presented through their representation. As Harry describes how to read the representation, many of his classmates exclaim "Oh!" while others object. Apparently, some of the students have not understood that each color shows $\frac{3}{4}$, even if $\frac{1}{4}$ appears in the first circle and $\frac{2}{4}$ appears in the second. As Harry finishes, some students say, "I get it!" However, this is not the end of the discussion. The teacher isn't convinced that everyone in the class can see how the representation displays each component of the problem. She invites the class to pose questions for Harry and Jay.

For the next six minutes, individual students explain what they see in the representation while the teacher asks the class a series of questions: "What is this [circle] one of?" "Where's the 12 days in this picture?" "Where is multiplication in a picture like this?" "How many groups of $\frac{3}{4}$ are there?" Periodically, students comment on how Harry and Jay's representation offers new insights for them.

In the second part, Isabel presents her solution strategy. However, as she stands before the class, at first she doesn't remember the steps of her own reasoning. In this 8-minute discussion, Isabel, the teacher, and the class work together to reconstruct Isabel's reasoning. By the end, Isabel recalls the steps she had taken to solve the problems, and her classmates have now worked through the reasoning, as well: Maya ran a total of 6 days. Since K.C. ran half of Maya's distance each day, he ran a total of 3 miles. And since Joey ran three times K.C.'s distance each day, he ran a total of 9 miles.

The culture already present in this classroom is that students are expected to share their thinking and respond to the ideas of their classmates. In fact, several of the students express pleasure in viewing the mathematics in new ways. Such a classroom environment is essential to students' engagement with MP3: *Construct viable arguments and critique the reasoning of others*.

² Unpublished video from the project, *Using Routines as an Instructional Tool for Developing Elementary Students' Conceptions of Proof*. © TERC, 2013. Used with permission. All rights reserved.